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The SUSY-GRACE system

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Abstract

We introduce a new method to treat Majorana fermions on the GRACE system which has been developed for the computation of the matrix elements for the processes of the standard model. In the standard model, we already have such particles as Dirac fermions, gauge bosons and scalar bosons in the system. On the other hand, in the SUSY models there are Majorana fermions. In the first instance, we have constructed a system for the automatic computation of cross-sections for the processes of the SUSY QED. It is remarkable that our system is also applicable to

another model including Majorana fermions (*e.g.* MSSM) once the definition of the model file is given.

1 Introduction

It has been widely accepted that there exists a symmetry called supersymmetry (SUSY) between bosons and fermions at the unification-energy scale. SUSY, however, is broken at the electroweak-energy scale. The relic of SUSY is expected to remain as a rich spectrum of SUSY particles, partners of usual matter fermions, gauge bosons and Higgs scalars, named sfermions, gauginos and higgsinos, respectively [1, 2, 3].

Since there exist so many particles and their interactions, it is a skilled job to calculate the cross-sections for the processes with the final 3-body or more. We have already known within the standard model that the numerical calculation of the helicity amplitudes with the program package CHANEL [4] is more advantageous to such a case than that of the traces for the gamma matrices with REDUCE.

It, however, is also hard work to construct a program with many subroutine calls of CHANEL. Thus we need a more convenient way to carry out such a work. The GRACE system [5], which automatically generates the source code for CHANEL, is one of the solutions. The system also includes the interface and the library of CHANEL, and the multi-dimensional integration package BASES [6].

The GRACE system has been developed for the computation of the matrix elements for the processes of the standard model. In the standard model, we already have such particles as Dirac fermions, gauge bosons and scalar bosons in the system. On the other hand, in the SUSY models there are Majorana fermions as mixed states of neutral gauginos and higgsinos.

Thus we are able to compute the SUSY processes with an automatic system, provided that we develop an algorithm to treat Majorana fermions in the system. The aim of this work is to construct such a system and test it with REDUCE. Since anti-particles of Majorana fermions are themselves, there exists so-called ‘Majorana-flip’, the transition between particle and anti-particle. This is the most important problem which we should solve when we realize the automatic system for computation of the SUSY processes.

2 SUSY-CHANEL into new GRACE

The method of computation in the program package CHANEL is as follows:

1. To divide a helicity amplitude into vertex amplitudes.
2. To calculate each vertex amplitude numerically as a complex number.
3. To reconstruct of them with the polarization sum, and calculate the helicity amplitudes numerically.

The merit of this method is that the extension of the package is easy, and that each vertex can be defined only by the type of concerned particles.

Here we propose an algorithm for the implementation of the embedding Majorana fermions in CHANEL as follows:

- **policy**

1. To calculate a helicity amplitude numerically, and square the absolute value of it.
2. To replace each propagator by wave functions, and calculate vertex amplitudes.
3. **Not to** move charge-conjugation matrices.

- **method**

1. To choose a direction on a fermion line.
2. To put wave functions, vertices and propagators along the direction in such a way:
 - i) To take the transpose for the reverse direction of fermions
 - ii) To use the propagator with the charge-conjugation matrix for the Majorana-flipped one.

As a result, the kinds of the Dirac-Majorana-scalar vertices are limited to four types:

- (1) $\overline{U}\Gamma U$
- (2) $U^T\Gamma\overline{U}^T$
- (3) $\overline{U}C^T\Gamma^T\overline{U}^T$
- (4) $U^T\Gamma^TC^{-1}U$

where U 's denote wave functions symbolically without their indices, and C is the charge-conjugation matrix. The symbol Γ stands for the scalar vertex such as

$$\Gamma = A_L \cdot \frac{1 - \gamma}{2} + A_R \cdot \frac{1 + \gamma}{2} \quad .$$

The vertices (2)~(3) are related to the vertex (1) which is the same as the Dirac-Dirac-scalar vertex in the subroutine of CHANEL. Thus we can build three new subroutines for the added vertices.

On the other hand, the GRACE system becomes more flexible for the extension in the new version called ‘**grc**’ [7]. We have performed the installation of the subroutines above with the interface on the new GRACE system.

3 Numerical results for tests

At the start for the check of our system, we have written the model file of SUSY QED. In this case, there is only one Majorana fermion, photino. The tests have been done in such a manner:

Process	Number of diagrams	Comment	Reference
$e^-e^- \rightarrow$	$\tilde{e}_R^-\tilde{e}_R^-$	2	Majorana-flip in internal lines
	$\tilde{e}_L^-\tilde{e}_L^-$	2	
	$\tilde{e}_R^-\tilde{e}_L^-$	2	
$e^-e^+ \rightarrow$	$\tilde{e}_R^-\tilde{e}_R^+$	2	Including pair [8]
	$\tilde{e}_L^-\tilde{e}_L^+$	2	annihilation [8]
$e^-e^+ \rightarrow$	$\tilde{e}_R^+\tilde{e}_L^+$	1	Values are [8]
	$\tilde{e}_R^+\tilde{e}_L^-$	1	equal [8]
$e^-e^+ \rightarrow$	$\tilde{\gamma}\tilde{\gamma}$	4	F-B symmetric
$e^-e^+ \rightarrow$	$\tilde{\gamma}\tilde{\gamma}\gamma$	12	Final 3-body [9]

Table I. The list of the tested processes.

1. To calculate the differential cross-sections at a point of the phase space in the two methods with GRACE and REDUCE.
2. To calculate the differential cross-sections over the phase space with REDUCE.
3. To integrate the differential cross-sections over the phase space in the two methods with GRACE and REDUCE.

For the GRACE system, we can get the differential cross-sections and the scattered plots by one time of the integration step with BASES.

In Table I, the tested processes are shown as a list. It has been also confirmed that the cross-sections for the charge-conjugated processes $e^-e^- \rightarrow \tilde{e}^-\tilde{e}^-$ and $e^+e^+ \rightarrow \tilde{e}^+\tilde{e}^+$ are identical. The references in the table are not the results of the tests, but for help. Hereafter the typical results are presented at the JLC-I energy. The masses are $M_{\tilde{\gamma}} = 50$ GeV, $M_{\tilde{e}_R} = 60$ GeV and $M_{\tilde{e}_L} = 70$ GeV.

In Fig. 1, we show the angular distribution of the process $e^\mp e^\mp \rightarrow \tilde{e}_L^\mp \tilde{e}_L^\mp$. In this case, there exist two Feynman diagrams with Majorana-flip in the internal lines. Results from two methods exactly coincide.

In Fig. 2, we show the angular distribution of the process $e^-e^+ \rightarrow \tilde{\gamma}\tilde{\gamma}$. In this case, there exist four Feynman diagrams with Majorana-flip in the external lines, and with both selectrons (\tilde{e}_R and \tilde{e}_L) in the internal lines. Here we use BASES for the calculation from the REDUCE output. The result is in beautiful agreement with the value that is obtained by GRACE at each bin of the histogram.

4 Summary

We introduce a new method to treat Majorana fermions on the GRACE system for the automatic computation of the matrix elements for the processes of the SUSY models. In the first instance, we have constructed the system for the processes of the SUSY QED because we should test our algorithm with the simplest case. The numerical results convince us that our algorithm is correct.

It is remarkable that our system is also applicable to another model including Majorana fermions (*e.g.* MSSM) once the definition of the model file is given. We should compute the single-photon event [9] and the resultant single-electron (positron) event from the single-selectron production [10] as soon as possible. It should be emphasized that the GRACE system including SUSY particles is the powerful tool for the purpose.

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